The Ultimate Guide to Bar Modelling

Understand and apply the bar model from basic arithmetic to multi-step word problems

KS1 / KS2

THIRD SPACE LEARNING

Contents

Where does bar modelling come from?	page 2
What is a bar model?	page 2
What are the benefits of using the bar model?	page 3
What are the different types of bar model?	page 3
How to introduce bar modelling into your school	page 7
Using the bar model across the Maths curriculum	page 10

Where does bar modelling come from?

Bar modelling is a method of representing Maths problems pictorially; it has been popularised by the Singapore Maths teaching method which is being used increasingly in the UK.

In the 1960s Jerome Bruner proposed that people learn in three stages: *concrete, pictorial, abstract*. Bar models are just one way of supporting children with the middle stage: *pictorial*.

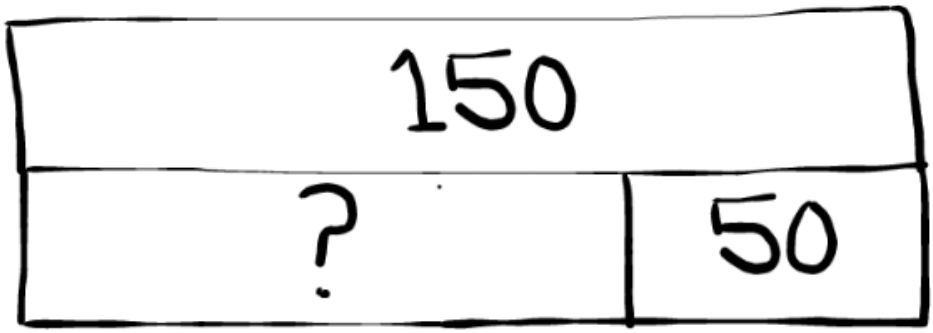
What is a bar model?

Also known as tape diagrams (in Japan) and strip diagrams (in the US), bar models are beginning to be used all over the world to help children to visualise Maths problems.

Put simply, a bar model is a diagram; the purpose of which is to represent a mathematical problem. Especially one which might be quite difficult to solve without visualising it first. There are several permutations of the bar model, but all follow the same basic principles:

- Bar models are constructed in a specific way, using rectangles to represent known and unknown amounts
- Bar models are designed to bring together all the relevant information from a problem, to contextualise any facts presented in a question. This helps children to begin working towards an answer

An example of a simple 'part/whole' bar model used to represent the equation: 150 – 50 = ?



It is important to note that bar modelling is not a method of calculation. In actual fact, bar models are primarily used as a way to help children decide which operation to use (the pictorial part of Bruner's learning stages).

Once a bar model has been constructed, learners can then complete the abstract phase: the calculation – this will be done using the child's preferred written or mental calculation methods.

What are the benefits of using the bar model?

Bar modelling helps children with one of the most essential parts to the learning process: visualising and making sense of a problem so they can then begin to move on to the more abstract part of calculating an answer. You can think of it as a tool to help pupils 'eke' out the steps and operations needed to solve the problem.

Once children are confidently able to use bar models as part of their reasoning process, **they can apply the technique in almost all areas of the Maths curriculum**.

Whilst there are many other ways of drawing diagrams to represent Maths problems, bar modelling is a simple, almost universal technique.

Bar modelling **does not require children to have equipment other than pencil and paper** making it very versatile. For example, bar models can be used in the classroom and in tests when the provision of other equipment it prohibited.

One of the key strengths of bar modelling - and the reason for its popularity - is that bar models can be used in a wide variety of mathematical topics.

The use of bar modelling **can help teachers to avoid teaching tricks which don't really make clear why or how the Maths works**.

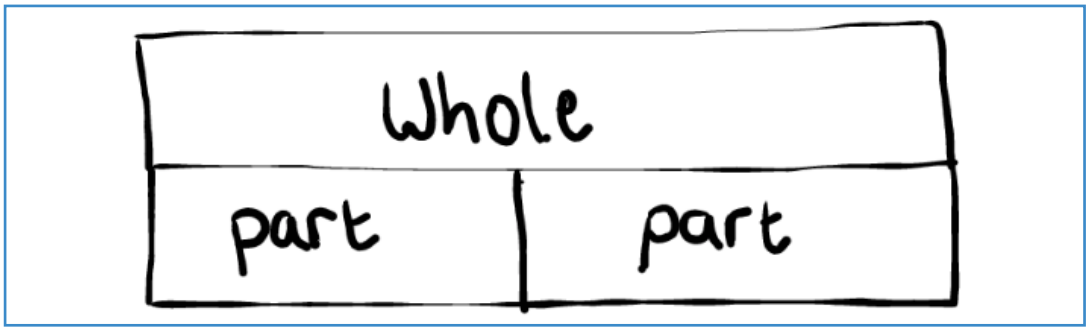
When a bar model is constructed to represent a problem it can then be manipulated, or worked on, to give a visual picture of what is happening when, for example, a calculation is performed. This understanding of the why and the how is essential for children when it comes to fluently applying their skills and knowledge to a wide range of mathematical problems.

What are the different types of bar model?

There are two main types of bar model, 'part/whole' and 'comparison':

Part/whole bar models

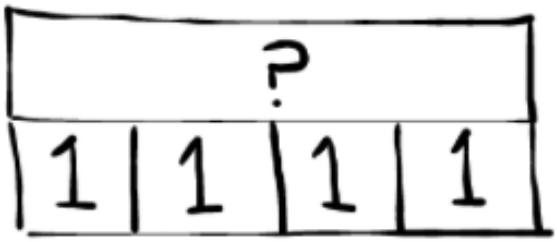
Part/whole bar models are made up of parts and wholes, where the whole represents the sum of the parts.



You can represent this in two ways:

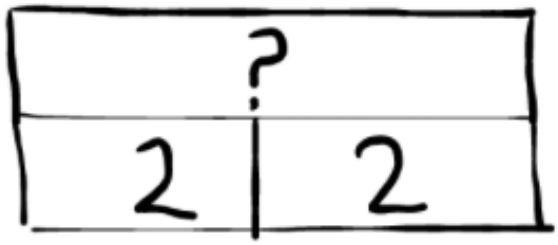
1. **As a discrete part/whole model**: The discrete model shows every unit as an individual box (as would be represented by cubes)

An example of a discrete part/whole bar model



2. **A continuous part/whole bar model**: This represents all amounts as one rectangle (e.g. 26kg is drawn as one bar and not 26 individual parts of a bar)

An example of a continuous part/whole bar model



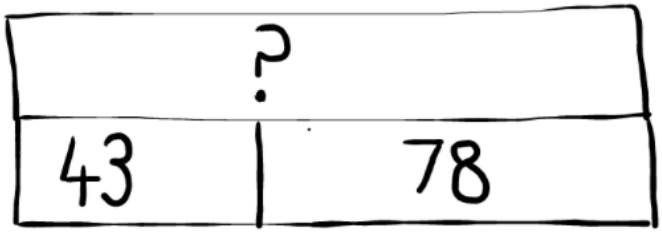
When drawing a part/whole bar model, make sure all rectangles are proportional to each other. For example, a rectangle representing 6 should be roughly twice the length of a rectangle representing 3.

This type of bar model is frequently used to represent; all four operations (including calculations involving more than one pair of numbers), fractions, measure, algebra, ratio and proportion and much more.

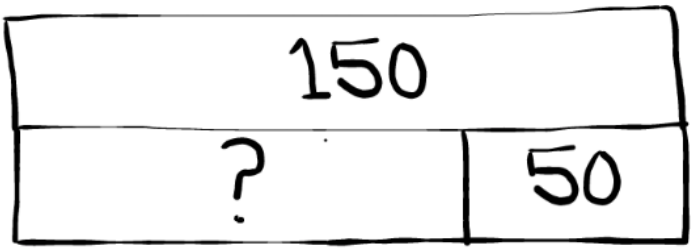
When using part/whole bar models, take care that they are only applied where helpful, rather than broadly attached to every area of the Maths curriculum. You should also make sure that pupils do not use them as a crutch, or become dependent upon them.

Here are some typical representations of bar models with the four operations:

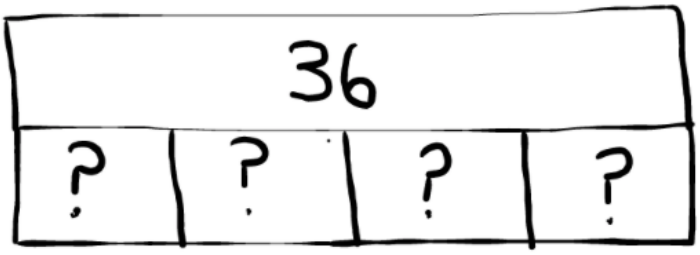
Bar model representing the addition equation '43 + 78 = ?'



Bar model representing the subtraction equation '150 – 50 = ?'



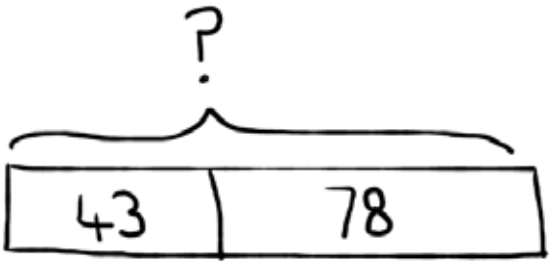
Bar model representing the division equation '36 ÷ 4 = ?'
or the multiplication' equation '4 x ? = 36'



With all these bar models, the use of the question mark is important. If children consider where their answer(s) should appear on their diagram (the unknown quantity, marked '?') then they are thinking about what the question is asking them to find out.

Some representations of part/whole bar models replace the 'whole' bar with a bracket, as below:

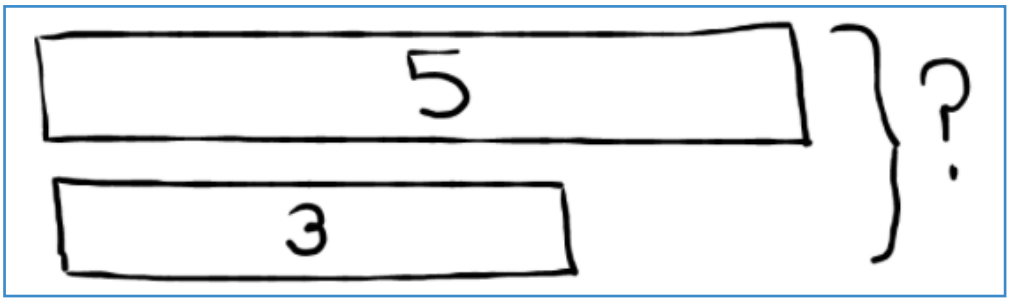
A bar model where the 'whole' bar has been replaced with a bracket



Comparison bar models

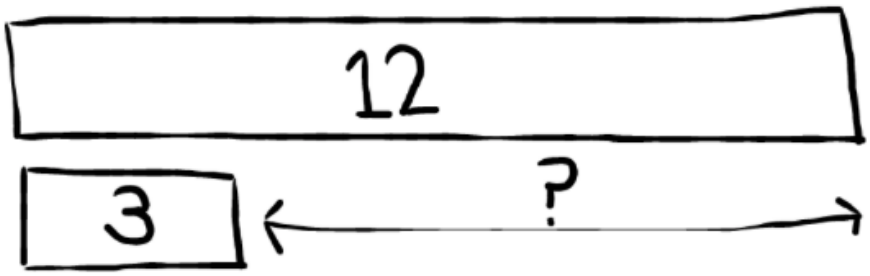
Rather than having a bar which represents the 'whole' in a problem, in a comparison bar model two or more vertically aligned bars are drawn to help children to compare two (or more) amounts.

With this type of bar model, brackets are often used to indicate the whole amount, as below:



Comparison models are particularly useful when finding differences between amounts, helping to reinforce the idea of using subtraction to find the difference. Here is an example of a problem and bar model representation:

Sandi has 12 football cards and Umar has 3. How many more cards does Sandi have than Umar



Summary

- **There are two main types of bar models**, the part/whole bar model, and the comparison bar model
- **Part whole bar models are made up of parts and wholes, where the whole represents the sum of the parts**. They are often used to represent problems involving all four operations, fractions, measure and more
- **Comparison bar models, where two or more vertically aligned bars are drawn** are generally used to find differences between amounts

How to introduce bar modelling into your school

Asking teachers to do something new always needs to be thought through carefully. Here are our list of recommendations to ensure bar modelling makes the best possible impact across your school.

- **Provide initial training on the method** in order to get staff on board and confident with bar models. The best way to do this is to ensure they really understand what bar modelling is all about. Consider introducing it in your next staff meeting. A simple Powerpoint, such as our presentation, *Representing Problems with Bar Modelling* can be a quick (and easy!) way to help staff get to grips with bar modelling.
- **Insist that teachers and children use the method as much as they can initially**, although you should still aim to get to a point where children can choose to use the technique only if they need it. This will need monitoring by the Maths leader and other senior leaders – new initiatives can often fall by the wayside if this doesn't happen.
- **Encourage teachers to be creative with their use of bar models** – challenge them, and the children, to use them across the Maths curriculum (for more on this, see the section entitled **Using bar models across the Maths curriculum**). This must be done judicially of course, it's not suitable for everything, rather it's about encouraging children to make reasoned choices about when bar modelling is best to use.
- **Make it clear that bar modelling will be used most successfully by children when it becomes a whole school approach** and that the younger they start, the better (within reason!).

When is the best time to introduce bar modelling?

If you've decided to introduce bar modelling in your school, the next question is often 'when?'

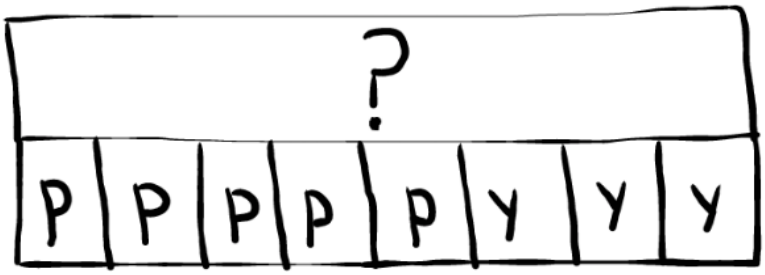
Bar modelling can be introduced to very young children – the younger they are when they first use it, the more useful they will find the technique as they grow older and come across more complex Maths problems.

In fact, without knowing it, children in the Early Years and in Key Stage 1 often use bar models in the form of cubes.

Let's look at the equation '5 + 3 = ?'

The cubes above are essentially a discrete bar model represented in concrete form. This can easily be transferred to *pictorial*. See below:

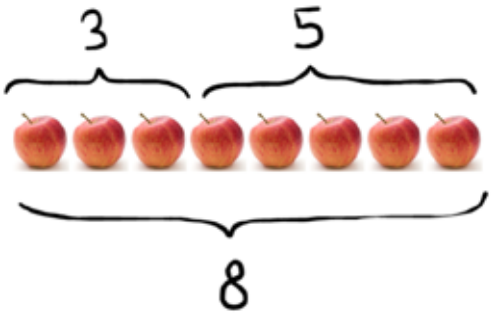
A discrete part/whole bar model representing the equation '5 + 3 = ?'



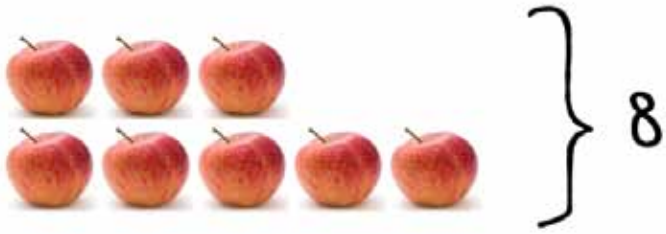
Many other concrete representations should be used with younger children – items such as fruit and toys can be used to introduce children to bar modelling.

See below for two types of example:

A part/whole bar model using fruit to represent the equation '5 + 3 = 8'



A comparison bar model using fruit to represent the equation '5 + 3 = 8'



What might progression in pupils' use of the bar model look like?

How pupils represent problems with the bar model will vary depending on their year group, but if you introduce the method effectively from the ground up, these are some key progression markers you might expect to see:

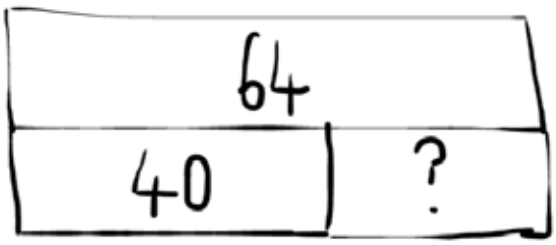
- **During Early Years (where appropriate) and in Key Stage 1**, children become confident with representing simple Maths problems - including word problems - using objects (the concrete stage).
- **By the end of Year 2** all children should be able to consistently represent a one-step problem with a bar model. Their use of the bar model can extend to multiplication and division, finding fractions and so on.
- **As children move into Key Stage 2**, the bar models they construct will remain much the same but they will learn to apply the bar model across the broader Maths curriculum and use a series of simple bar models to answer multi-step problems (for more on this read our blog: *How to Use Bar Models to Solve Multi-Step SATs Problems)*.

The importance of language when introducing bar models

The language of 'part' and 'whole' should be used consistently for a good success rate with bar models. All one-step Maths problems fall into one of the following categories:

- Whole unknown
- Number of parts unknown
- Size of parts unknown

In this bar model, the bar labelled 64 would be referred to as the 'whole' and the size of a part (?) is unknown.



Addition and multiplication problems are 'whole unknown' problems whereas subtraction problems are 'size of part unknown'. Division problems can either be 'size of part unknown' or 'number of parts unknown'.

Once missing number problems are introduced, things will change. For example, 10 + ? = 12 is a 'size of part unknown'. Using this language early on will help children when they come to more complex word problems later on in their school life (see Nick Hart's post *Throwing Out That Old RUCSAC* for more on problem solving approaches).

Summary

- **Familiarise staff with the key concepts of bar modelling through initial training** and encourage them to use the method across different year groups and different Maths topics.
- **Make sure children are familiar with concrete resources first** before introducing them to any kind of bar model
- **Introduce bar models as early as possible (within reason!)** - starting early helps children understand the structure of problem solving questions and decide what the question is asking them to find out
- **Use terminology consistently across your school** - always refer to the 'parts', 'whole' and emphasise the 'knowns' and 'unknowns' in different problem types

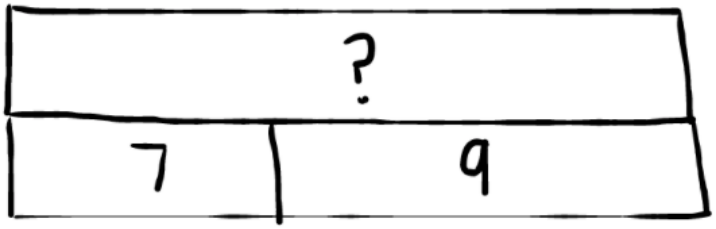
Using the bar model across the Maths curriculum

As we've mentioned bar modelling can be used across a wide range of mathematical topics. Here are some examples of how to use bar models in specific contexts, and the best ways to ensure success.

Missing Number Problems

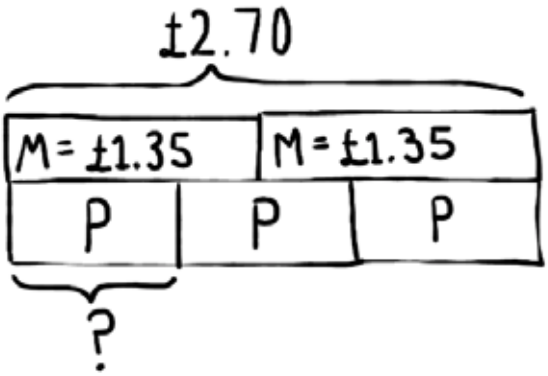
Missing number problems feature in the curriculum from Year 1 to Year 6. Bar models are a great way to help children of all ages to find missing numbers, whether that's 7 = ? – 9 in Year 1 or expressing missing number problems algebraically in Year 6.

A bar model to represent the problem 7 = ? – 9



Take question 14 from the **2017 SATs Reasoning Paper 3,** for example:

3 pineapples cost the same as 2 mangoes. One mango costs £1.35.
How much does one pineapple cost?



Using the part/whole bar model in this way, allows children to see that 3p = 2m. Children now know, and can therefore complete, the calculation (written or mental) alongside their bar model.

Fractions problems

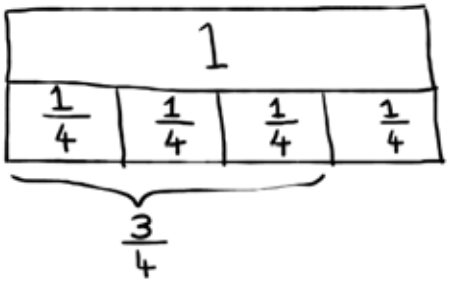
Fractions are mentioned 104 times in the National Curriculum so they're quite a big deal.

Modelling fractions using bar models is one of the most intuitive ways of showing fractions pictorially. It might even be that children will gain a greater understanding of bar modelling in general, giving them skills which are transferable to other areas of the Maths curriculum.

As such, there are a wide range of contexts that a bar model can be used to represent fractions problems.

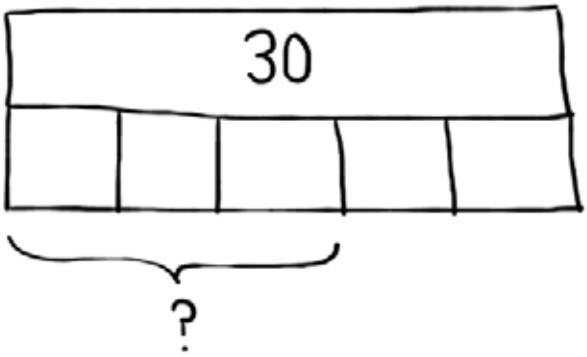
In Year 2 children have to recognise, find, name and write fractions, such as $\frac{3}{4}$ which can be represented as below:

A bar model representing three quarters



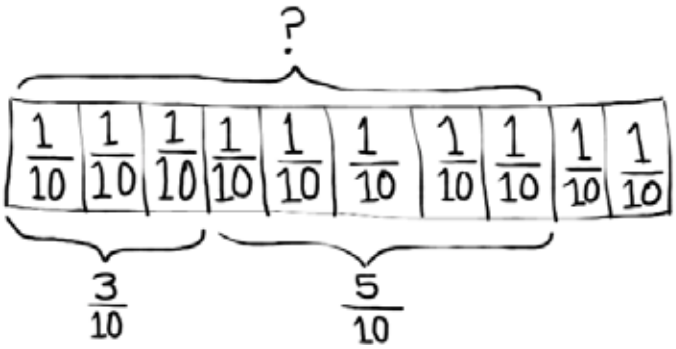
Also, bar models can support pupils to find fractions of amounts, as in the equation below:

A bar model representing the equation: What is $\frac{3}{5}$ of 30?



In Year 3 children must add fractions with the same denominator:

A bar model used to represent the equation $\frac{3}{10} + \frac{5}{10} = ?$



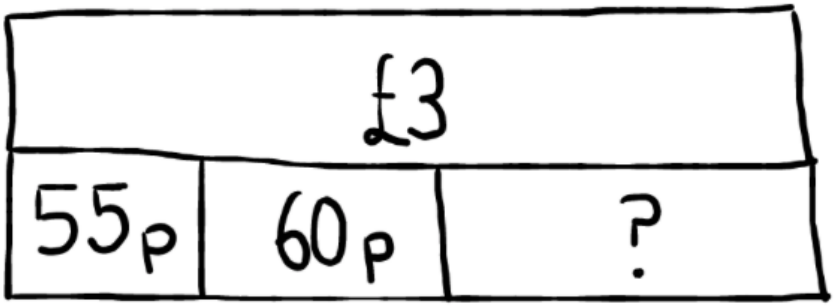
It is worth noting, however, that some of the Year 6 fraction content is not best modelled using bars. For instance, area models can be used to represent multiplication of fractions more naturally than bar models.

Money problems

Whilst everything that's done with money is to do with one of the four operations, it's worth thinking about how bar modelling can help children to visualise the Maths in money problems.

This starts in Year 2 with addition and subtraction and gradually gets more difficult. By Year 5 children are asked to use the four operations to solve complex problems involving money, using decimal notation. Let's look at a word problem:

A boy has £3. He buys some crisps for 55p and a chocolate bar for 60p. How much change does he get?



Bar models like this can help children to understand key (and often confusing) concepts in money problems. For example, by using a bar model children should be able to see that:

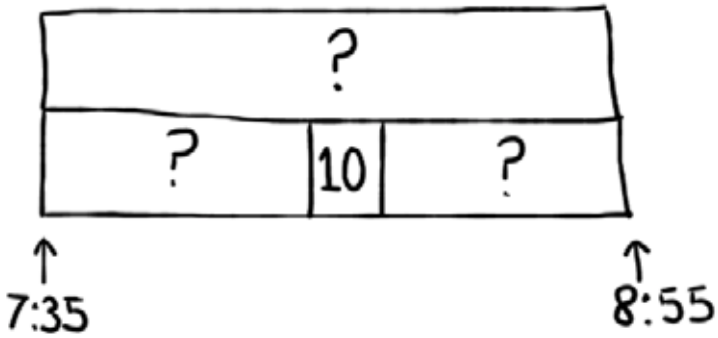
- When someone buys more than one thing the costs must be added
- When someone spends money subtraction is happening
- When sharing money equally division is happening
- When sharing money unequally there are multiple steps of subtraction

The language used in problems like these often confuses children but bar models can help them to visualise the Maths behind the words – acting out problems are a great precursor or accompaniment to bar modelling money problems.

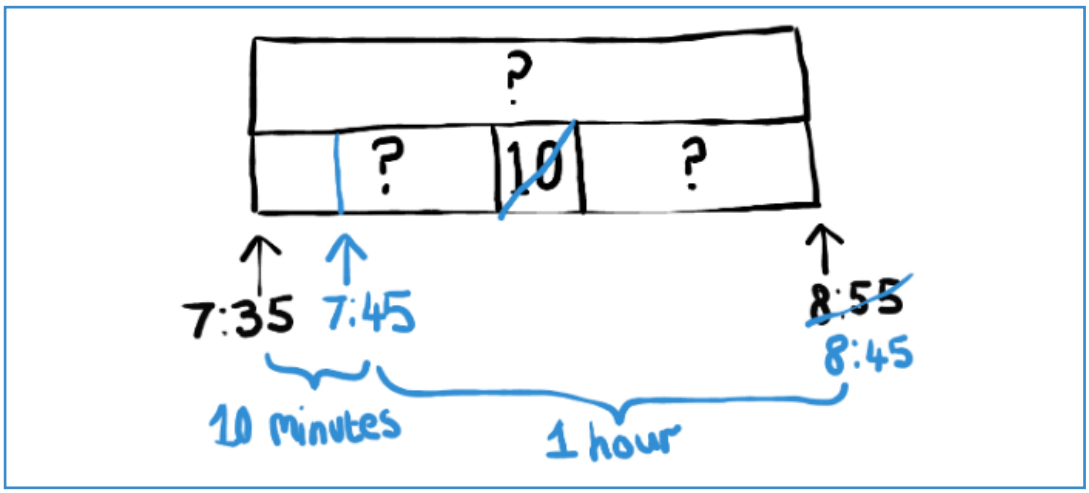
Time problems

Time can be a tricky thing for children to visualise as their go-to pictorial model is a clock which goes round and round. As far as we know time doesn't go round, it moves forward in a line so bar models can represent some time problems quite well. Here's an example:

A film starts at 7:35 pm and ends at 8:55 pm. There is an ice cream break of 10 minutes halfway through. How long is the film?



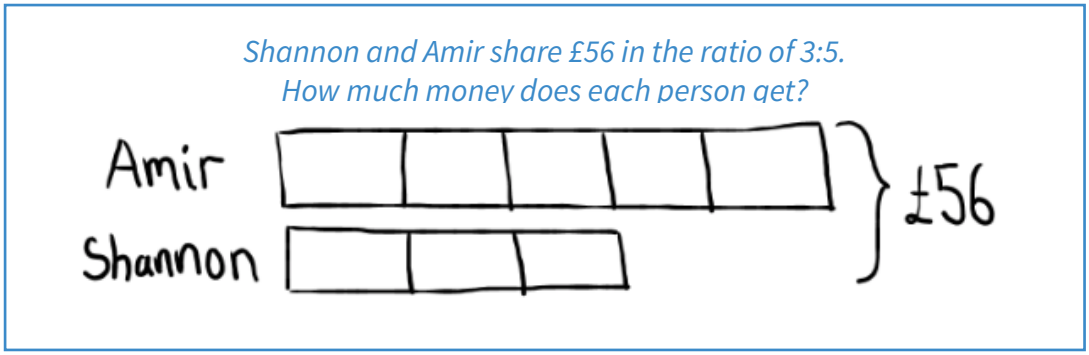
Children can use bar models like this as a working document, changing parts of it as they work out their answers:



In more complex multi-step problems like this it is important that children know which of the unknowns on their bar model is the one that will give them an answer. In this case none of the unknowns on the original diagram actually provide the answer.

Ratio problems

The comparison bar model is a gift when it comes to ratio problems, which is particularly significant given that in the 2017 KS2 tests the ratio question was one of the most poorly answered.



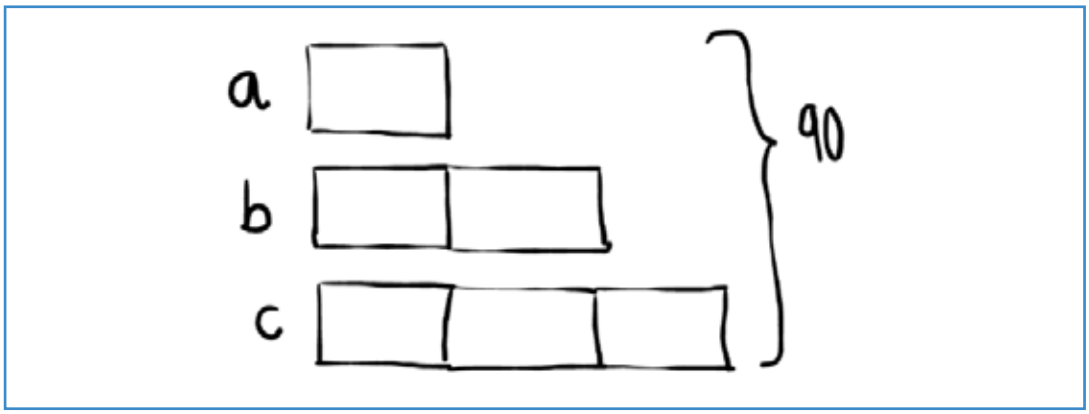
By representing this problem with a bar model, it makes it much easier to see that the whole amount is made of 8 equal parts (3 + 5). Therefore the £56 needs to be first divided by 8 (good knowledge of times tables comes in handy here!) before then being multiplied by the number of parts of £56 each person gets.

Here's another example of a problem which uses a multi-part ratio:

90 sweets are shared between three bowls (a, b and c) in the ratio of 1:2:3. How many more sweets does bowl b have than bowl a?

This problem could easily be presented without reference to ratio:

90 sweets are shared between bowls a, b and c. Bowl b contains twice the amount that bowl a contains. Bowl c contains three times the amount that bowl a contains. How many more sweets does bowl b have than bowl a?



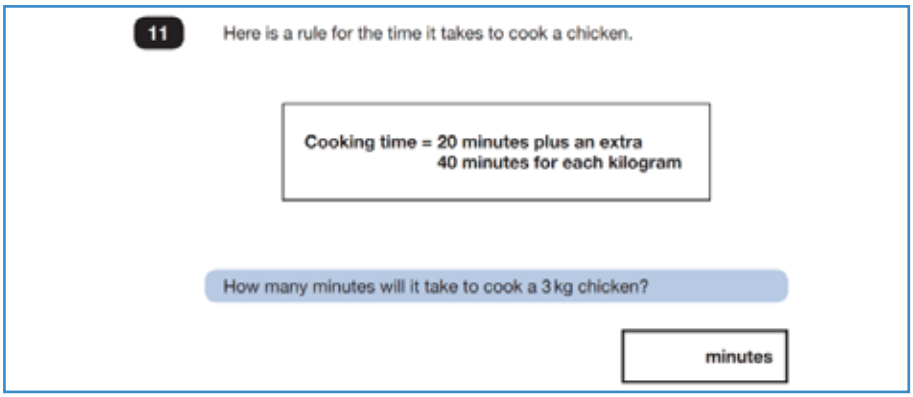
Again, pupils can clearly see that 6 equal parts make the whole (90) and so division is needed.

It's worth pointing out that these bar models give more information than the question asks for – children must be careful only to answer what has been asked, in this case, 'How many more sweets does bowl b have than bowl a?'

Unfamiliar problems

When the following problem (question 11) arrived in the **2017 SATs Reasoning Paper 2** it wasn't exactly well received; teachers thought the context was too far outside of a child's range of experience. However, if children have been exposed to complex problems in a variety of contexts, they have a good chance of being able to work out what they need to do.

Bar modelling can help with this, especially if they are familiar with using the technique to help them visualise problems.



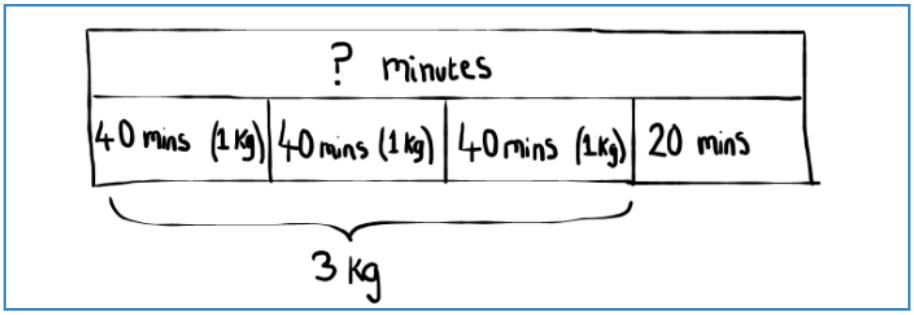
11 Here is a rule for the time it takes to cook a chicken.

Cooking time = 20 minutes plus an extra 40 minutes for each kilogram

How many minutes will it take to cook a 3 kg chicken?

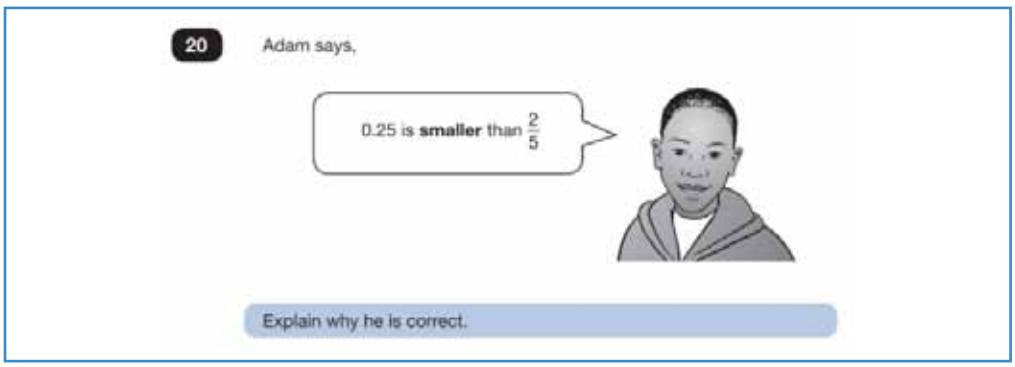
minutes

A bar model (another example of a time bar model) can make this much simpler:



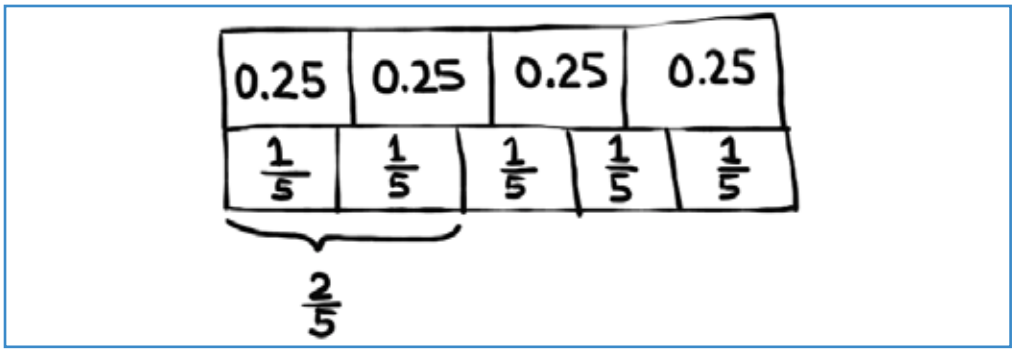
Giving explanations

One thing that many teachers value is the ability to explain mathematical understanding. The national tests often have a question which requires some written explanation of why something is right or wrong. Indeed the National Curriculum aims to ensure that children can justify and prove their mathematical understanding. Yet sometimes it can be tricky for children to explain what they know. Drawing and describing a bar model can help them to develop the language they need to talk about a problem.



For this problem (question 20 from the **2017 SATs Reasoning Paper 2**) children can model the decimal and the fraction in a comparison model and then use their drawings to help with a written explanation. To draw this children will need to have prior knowledge about how many lots of 0.25 are in a whole, and how many fifths are in a whole.

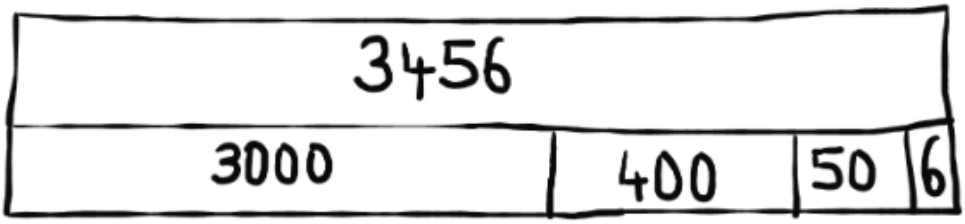
For a child who doesn't immediately equate $\frac{1}{5}$ with 0.2, the diagram they draw should prompt them to see that the whole is split into 5 equal parts and these parts could also be written as decimals in order to compare the fractions to the decimals.



Place value problems

When partitioning numbers to understand place value a bar model can be used to show the different parts.

An example of how a part/whole bar model can be used to partition numbers



Bar models don't have to be totally accurate in their representations of the relative sizes of numbers, but it is important they at least show that larger numbers are in some way proportionally different in size to smaller numbers. Given that place value is usually covered at the beginning of the year, using bar models at that point is a good way to get children practising their representations of different number sizes.

Summary

- **Bar modelling is an incredibly flexible approach** which pupils can use to represent problems involving fractions, missing numbers, time, ratio, money and place value
- **Drawing a bar model can be a great way to support pupils who struggle to 'talk' about maths** as it gives them a visual on which to base their explanations
- **Make sure pupils can recognise when to use a bar model** and when a different method might be more helpful (for example, when multiplying fractions in Year 6)

Conclusion

Bar modelling is, quite rightly, taking the world by storm. Though every school is different, we believe that bar modelling can be applied in almost any context. It's impact is greatest in those schools that have really managed to immerse teachers and pupils in the bar modelling approach.

We hope this guide has helped you understand a little bit more about bar modelling and how to embed the technique in your school. Happy modelling!